Comment II on ''Simple measure for complexity''

P.-M. Binder*
Departamento de Física, Universidad de Los Andes, Apartado Aéreo 4976, Bogotá, Colombia

Nicolás Perry
Department of Physics, Duke University, Durham, North Carolina 27708
(Received 26 July 1999)

The measure of complexity recently proposed by Shiner, Davidson, and Landsberg [Phys. Rev. E **59**, 1459 (1999)] does not adequately describe the transition from regular to indexed languages observed at the period-doubling accumulation points of quadratic maps. This Comment points to a generic inadequacy of that measure.

PACS number(s): 05.20.−y, 05.90.+m

Shiner, Davidson, and Landsberg [1] (henceforth SDL) introduced a complexity measure $\Gamma_{\alpha\beta}$, in which a term that quantifies order multiplies another term that quantifies disorder. This measure has the attractive feature that different functional dependences of complexity vs measure of disorder are possible [2]. However, in this Comment we would like to point out a shortcoming of the new measure. Figure 5 of SDL shows all the possible functional forms of complexity vs disorder of their measure. None of them allows for a finite-entropy singularity at the accumulation points of period-doubling bifurcations, as shown, for example, in Ref. [3]. This singularity indicates the existence at these points of a class of languages not describable by deterministic finite automata, and corresponds roughly to the well-known ''edge of chaos'' [4].

Equation (6) in SDL confirms that $\Gamma_{\alpha\beta}$ has a *finite* maximum, which is exemplified for the particular case $\alpha=\beta=1$ in Fig. 3 of SDL. Nevertheless, the period-doubling accumulation points are described by an indexed language [3,5,6] for which the regular-language complexity is infinite. More precisely, it has been recently shown [7] that the appropriate automaton needed to recognize the symbolic dynamics at these points requires memory of two stacks, or one queue. It can be seen in Fig. 3 of SDL that not even the maxima of their measure correspond to instances of an indexed language in the logistic map. We therefore think that the statement in Sec. III of SDL that their measure ''behaves similarly to the effective measure complexity of Grassberger'' only holds on the surface. Certainly Grassberger's measure [8] will pick up the nonregularity of a language, as will Crutchfield and Young's measure [3]. This can be (and has been) done by calculating both of these measures over several word lengths (Grassberger) or tree depths (Crutchfield) at the points in question, and noticing their lack of convergence to a finite value; see also the discussion in p. 461 of Ref. [9].

A good measure of complexity should be able to discern at least *some* classes of complexity in the extended Chomsky language hierarchy [10]. Even in the cases in which a measure works in a different way than what is expected [11,12], it should be well understood why: see the discussion in p. 256 of Ref. [6] for this example. Given that the measure $\Gamma_{\alpha\beta}$ proposed in SDL is a simple function of disorder (entropy), it is clear to us that it cannot probe carefully the information-processing capabilities of the systems being examined [13]. This seriously limits the usefulness of their new measure, which cannot distinguish between regular and *any* class of higher-ranked languages in the hierarchy. The latter include most chaotic systems, which have no Markov partitions and are therefore not regular (see Ref. [6], pp. 202 and 203, Ref. [11], and Ref. [14]), most spatially extended systems, which can occupy any level in the language hierarchy [6] including universal Turing machines [15], and D0L (substitutive) languages [16], of interest in biology, among others. For these reasons, we urge potential users of the measure proposed in Ref. [1] to carefully interpret their results, and to complement them with other well-tested measures, such as those given in Refs. [3,6,11,17].

*Corresponding author. Electronic address: p@faoa.uniandes.edu.co

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